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Appendix

The Principles and Standards For School Mathematics National Council of Teachers of Mathematics

STANDARD 1 Number and Operation 1. Understand numbers, ways of representing numbers, relationships among numbers and number systems	PRE-K – 8 count fluently with understanding and recognize “how many” in small sets of objects understand the cardinal and ordinal meaning of numbers in quantifying, measuring, and identifying the order of objects connect number words, the quantities they represent, numerals, and written words, and represent numerical situations with each of these develop an understanding of the relative magnitude of numbers and make connections between the size of cardinal numbers and the counting sequence develop an understanding of the multiple relationships among whole numbers by comparing, ordering, estimating, composing, decomposing, and grouping numbers, including beginning understandings of place value understand and represent familiar fractions, such as $\frac{1}{2}$ and $\frac{3}{4}$	3-5 understand the structure of the base ten number system, including representations of decimals read and write large numbers, and relate notation to the meaning of these numbers develop meaning for fractions as parts of a unit whole, as a part of a collection, as numbers, and as a division of whole numbers continue to develop a sense of the relative magnitude of numbers, with a focus on powers of 10 (esp. hundreds and thousands) and their role as benchmarks in the number system develop meaning for factors and multiples, become fluent with factor and multiple relationships, and classify numbers according to their factor (prime, composite)	6 – 8 work flexibly with equivalent fractions, decimals, percents; compare and order these numbers efficiently and accurately, find their approximate locations on a number line, and choose appropriate and convenient forms of these numbers for solving problems develop meaning for integers, and be able to represent, compare and order them develop an understanding of large numbers, including the use of benchmarks to comprehend their magnitude; and recognize understand and appropriately use various representations for large numbers (e.g., exponential, scientific, and calculator notation) use number theory concepts (e.g., factors, multiple, prime factorization, relatively prime numbers) to solve problems and to understand ideas about rational numbers develop an understanding of the properties of the integer and rational number systems (e.g., order, density, and additive and multiplicative inverses)	9 - 12 increase their understanding of systems for representing numbers and quantities, including matrix representations for arrays of quantities compare and contrast properties of numbers and number systems begin to understand complex numbers as a superset of the real numbers and as a system containing solutions for equations that not solvable over the real numbers become familiar with finite sequences and series, including arithmetic and geometric examples, and develop an informal understanding of some infinite sequences and series, especially geometric series
			recognize and use commonly encountered irrational numbers (e.g., Π)	

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 1.1 continued		recognize and use common fraction, decimal, and percent equivalents		
		identify characteristics of particular numbers (e.g., odd/even, prime/composite) and use this information to describe characteristics of classes of number (e.g., multiples of 7)		
		develop strategies for judging the size of fractions and decimals and for comparing them, using a variety of models and of benchmarks (such as $1/2$ or $.5$)		
STANDARD 1 Number and Operation	understand different meanings of addition and subtraction of whole numbers and the relation between the two operations	understand the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array)	extend understanding of operations to include operations on fractions, decimals, percents, integers, and nonnegative whole number exponents	develop an understanding of the meaning of, and representations for, operations on vectors and matrices, and, with appropriate technology, be able to use these operations to solve systems of linear equations
2. Understand the meaning of operations and how they relate to each other	understand situations that lead to multiplication and division such as equal groupings of objects and sharing equally	identify and use relationships between operations to solve problems (e.g., multiplication as the inverse of division)	understand the effects of operating among fractions, decimals, percents, and integers	develop fluency operating on real and complex numbers, vectors and matrices, using by-hand operations for simple cases and using technology for more complex cases
	develop understanding about the effects of the operations on whole numbers	identify and use properties of operations to solve problems (e.g., 28×7 is equivalent to $(7 \times 20) + (7 \times 8)$)	recognize and use the properties of operations on integers and other rationals, such as closure, associative, commutative, and distributive properties understand and use the inverse relationships of addition and subtraction, multiplication and division, and square and square roots to solve problems extend understanding of counting to include elementary combinatorics	continue to develop an understanding of permutations and combinations as counting techniques in increasingly complex situations

STANDARD 1 Number and Operation 3. Use computational tools and strategies fluently and estimate appropriately	PRE-K – 8 develop and use strategies and algorithms to solve number problems	3-5 develop fluency with single-digit multiplication facts and their related division facts by grade 4, and use facts to efficiently compute related problems (e.g., 30×50 is related to 3×5 , 300×5 and 15×100)	6 – 8 develop, analyze and compare algorithms for computing with fractions, decimals, percents and integers, and become efficient and accurate in computing with them	9 - 12 analyze algorithms for operations with numbers, recognize some of the roles and limitations of particular algorithms, and be able to verify the viability of selected algorithms
	develop fluency with addition and subtraction facts by the end of second grade	develop and compare whole-number computational algorithms for each operation that are based on understanding number relationship, the base ten number system, and the properties of these operations, and by the end of grade 5 develop efficiency and accuracy in using algorithms	develop, analyze and compare methods for solving problems, involving proportions (e.g., scaling, finding equivalent ratios)	develop an understanding of the effects of measurement error on computed values
	compute using a variety of methods, including mental computation, paper and pencil, and calculators, and choose an appropriate method for the situation	develop and use computational estimation strategies based on understanding of number concepts properties and relationships	develop and refine strategies for estimating (including fractional quantities); use estimation as a means to check the reasonableness of results	develop the ability to distinguish between estimation and approximation, and use each appropriately in technological and nontechnological settings
	recognize whether numerical solutions are reasonable.	estimate sums and differences of common fractions and decimals using benchmarks (e.g., $3/8 + 1/3$ must be less than 1 since both fractions are less than $1/2$)	select and use appropriate methods for computing from among mental arithmetic, estimation, paper and pencil, and calculator, depending upon the situation at hand develop and use visual models, benchmarks and equivalents to add and subtract with common fractions (e.g., $1/2 + 1/4$ is the same as $1/4 + 1/4 + 1/4$ or $3/4$; or parts of a “pie”)	

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 1.3 continued			develop and use visual models, benchmarks and equivalents to develop computations, procedures for addition and subtraction of decimals (e.g., determining $0.10 + 0.15$ by shading in a 1- by 10 grid) choose appropriate computational procedures and tools (e.g., calculators, pencil/paper, mental computation) to solve problems.	
STANDARD 2 Patterns, Functions, and Algebra 1. Understand various types of patterns and functional relationship	sort and classify objects by different properties	identify, describe and extend geometric and numeric patterns, including growing and shrinking patterns	analyze, create, and generalize numeric and visual patterns, paying particular attention to patterns that have a recursive nature	recognize equivalent forms of an expression, equation, function, or relation
	order objects by size or other numerical property (seriation)	represent and record patterns, using tools such as tables and graphs	use patterns to solve mathematical and applied problems	be familiar with classes of functions, including linear, quadratic, power, polynomial, rational, absolute value, exponential, logarithmic, trigonometric, and step functions; understand piecewise-defined functions and their properties; analyze the effects of parameter changes; and describe local and global behavior
	identify, analyze and extend patterns and recognize the same pattern in different manifestations	identify and describe relationships between two quantities that vary together (e.g., the length of a square and its area)	represent a variety of relations and functions with table, graphs, verbal rules, and when possible, symbolic rules	select appropriate representations (numerical, graphical, verbal, and symbolic) for the functions and relations embedded in quantitative situations, convert flexibly among representations, interpret representations, and use them to interpret the situations represented

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 2.1 continued	describe how both repeating and growing patterns are generated	investigate and describe situations involving inverse relationships (e.g., the more friends, the fewer the cookies for each person, or the larger the denominator in a unit fraction, the smaller the quantity)		use a variety of symbolic representations, including recursive definitions and parametric equations, to explore the behavior of functions and relations
		identify, express, and verify generalizations and use them to make predictions (e.g., doubling a number, then doubling again is the same as multiplying by four)		reason (from graphs, tables, and formulas) about functions derived from other functions via transformation (e.g., $g(x)=f(x-2)+5$), inversions, composition, and arithmetic combination.
STANDARD 2 Patterns, Functions, and Algebra 2. Use symbolic forms to represent and analyze mathematical situations and structures	illustrate general principles (e.g., commutativity) using specific numbers	identify and use relationships between operations to solve problems (e.g., multiplication as the inverse of division)	develop a sound conceptual understanding of equation and of variable	represent situations that involve variable quantities with expressions, equations, inequalities, and systems of equations using a variety of equivalent forms
	understand reversal of operation	identify and use algebraic properties of operations to solve problems (e.g., 28×7 is equivalent to $(7 \times 20) + (7 \times 8)$)	explore relationships between symbolic expressions and graphs, paying particular attention to the horizontal and vertical intercepts, points of intersection, and slope (for linear relations)	develop fluency operating on polynomials, vectors, and matrices using by-hand operations for the simple cases and using technology for more complex cases
	use concrete, pictorial, and verbal representations of numerical situations, including invented notation	develop the concept of variable as a useful tool for representing unknown quantities	become fluent in generating equivalent expressions for simple algebraic expressions and in solving linear equations and inequalities	understand symbolic algebra as abstracted arithmetic
	use appropriate symbolic representation of mathematical situations	use variables (boxes, letters, or other symbols) to solve problems or to describe general rules	use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships	be able to explain, compare, and contrast the major properties of the objects and operations defines within and across systems (e.g., rational numbers, polynomials, matrices, and functions) as they follow certain rules or laws of structure

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 2.2 continued				develop strategies for deciding whether symbolic results generated with technological tools are reasonable, and interpret such results in meaningful ways
STANDARD 2 Patterns, Functions, and Algebra	make comparisons and describe change qualitatively (e.g., taller than)	represent and investigate how a change in one variable relates to the change in a second variable (e.g., the height of a plant over time)	model and solve contextualized problems using various representations, such as graphs and tables, to understand the purpose and utility of each representation	model a wide range of phenomena with a variety of functions, including linear, quadratic, exponential, rational, trigonometric, and recursively defined functions, and recognize that a particular type of function can model many different situations
3. Use mathematical models and analyze change in both real and abstract contexts	make comparisons and describe change quantitatively (e.g., 3 inches taller)	identify and describe situations with varying rates of change (e.g., a fund-raising effort brought in a small, steady amount of money in the beginning, but more each day as the deadline approached)	develop an initial understanding of rate of change, with emphasis on the connections among slope of a line, constant rate of change, and meaning in context	approximate and interpret accumulation and rates of change, both graphically and numerically, for functions representing a variety of situations.
STANDARD 3 Geometry and Spatial Sense	model concrete situations using addition and subtraction of whole numbers		explore different types of change occurring in discrete patterns, such as proportional and linear change	approximate and find intercepts, local extreme values, and asymptotic behavior of functions, and interpret such results in given contexts
1. Analyze characteristics and properties of two- and three-dimensional geometric objects	recognize, name, build, draw, describe, compare, and sort two- and three-dimensional shapes	identify, compare, and analyze attributes of two- and three-dimensional geometric figures	precisely describe, classify and compare types of plane and solid figures (e.g., angles, triangles, quadrilaterals, cylinders, cones, etc.) according to their main features	explore relationships among, make and test conjectures about, and solve problems involving classes of two- and three-dimensional geometric objects
	recognize and locate geometric shapes and structures in the world	classify two- and three-dimensional shapes according to their attributes, and develop definitions of classes of shapes (e.g., triangle, pyramid)	analyze and understand geometric relationships among two-dimensional and three-dimensional figures	connect geometry to other strands of mathematics (e.g., measurement, algebra, trigonometry), relate it to other areas of interest (e.g., art, architecture), and use it to solve problems

STANDARD 3.1 continued	PRE-K – 8 describe attributes and parts of two- and three-dimensional shapes	3-5 investigate, describe, and reason about the results of subdividing, combining, and transforming shapes using models and representations	6 – 8 use proportions to examine relationships between similar plane figures	9 - 12 recognize geometry as an example of a deductive system, built from undefined terms, axioms, definitions, and theorems; use deduction to establish the validity of geometric conjectures and to prove theorems		
					investigate and reason about properties of geometric figures (e.g., the number of diagonals of a regular polygon)	create and critique inductive and deductive arguments concerning geometric ideas and relationships
					build and use geometric vocabulary to describe two- and three-dimensional shapes	recognize and apply geometric ideas and relationships outside the mathematics classroom, in areas such as art, science, and everyday life
					understand and demonstrate congruence	
					make conjectures about properties and relationships of shapes and use spatial reasoning and geometric tools to test them	
					describe location and movement using common language and geometric vocabulary (i.e., location of an object in a room or on a grid)	learn to use coordinate geometry to display geometric relationships between related quantities, particularly when the relationship is linear
					set up a coordinate graph (include axes, origin, and scale), and use it to mark and read position using coordinate pairs	use coordinate geometry to represent and examine the properties of geometric figures, especially those containing sets of parallel or perpendicular lines
					make and use coordinate maps to represent actual places (e.g., post office, school) in an area	use relationships found in right triangles (e.g., the Pythagorean relation, isosceles right triangles, 30-60-90 degree triangles) to solve problems
					investigate and predict the results of putting together and taking apart shapes	
					recognize congruent and similar shapes	
relate geometric ideas to number and measurement ideas						
STANDARD 3 Geometry and Spatial Sense Select and use different 2. representationa 1 systems, including coordinate geometry and theory graph	describe, name, interpret, and apply ideas of relative position in space	describe location and movement using common language and geometric vocabulary (i.e., location of an object in a room or on a grid)	learn to use coordinate geometry to display geometric relationships between related quantities, particularly when the relationship is linear	investigate and verify conjectures and solve problems involving two- and three-dimensional figures represented with rectangular coordinates		
	describe, name, interpret, and apply ideas of direction and distance in navigating space	set up a coordinate graph (include axes, origin, and scale), and use it to mark and read position using coordinate pairs	use coordinate geometry to represent and examine the properties of geometric figures, especially those containing sets of parallel or perpendicular lines	explore other coordinate systems (e.g., navigational, polar, spherical) and their uses		

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 3.2 continued		represent rectangles and right triangles on a coordinate system and identify vertices with coordinates	explore the use of other representational systems, particularly networks	use trigonometric relationships to solve problems
		find distance between points along horizontal and vertical lines of a coordinate system		
STANDARD 3 Geometry and Spatial Sense 3. Recognize the usefulness of transformations and symmetry in analyzing mathematical situations	recognize and apply slides, flips, turns; predict the effects of transformations on shapes	given a distance, find pairs of points on the coordinate system separated by that distance		
	recognize and create reflectional and rotational symmetry of two- and three- dimensional objects	predict the results of sliding, flipping, and turning two-dimensional figures	describe size position and orientation of figures under informal transformations such as flips, turn, slides, and magnification	represent translations, reflections, rotations, and dilations/contractions of objects in the plane, using sketches, coordinates, vectors, or matrices, and use these representations to gain information about the transformations
		describe a motion or series of motions that will show that two figures are congruent	use line and rotational symmetry to describe and classify polygons and polyhedras	extend transformations to three dimensions, to include reflectional and rotational symmetry of solids
		identify and describe line and rotational symmetry in various two-dimensional shapes	understand the concepts of congruence and similarity using transformations	understand transformations (under the operation of composition) as an algebraic system of functions
	explore symmetry in three-dimensional objects	explore symmetry in three-dimensional objects	explore the composition of transformations (e.g., successive flips in different lines)	

STANDARD 3 Geometry and Spatial Sense 4. Use visualization and spatial reasoning to solve problems both within and outside of mathematics	PRE-K – 8 create mental images of geometric shapes (spatial memory and spatial visualization)	3-5 identify a three-dimensional shape from a set of two-dimensional views (e.g., match a solid with its front, side, and top views)	6 – 8 develop fluency with two-dimensional representations of three-dimensional objects	9 - 12 draw and interpret two- and three-dimensional objects, including those involving overlapping figures/objects and those requiring auxiliary lines
	determine and represent objects from different perspectives and points of view	visualize and sketch three-dimensional shapes in two dimensions (e.g., make views, nets, or isometric drawings)	compose and decompose two- and three-dimensional figures in order to solve problems	analyze cross-sections, truncations, and compositions/decompositions of three-dimensional objects
STANDARD 4 Measurement 1. Understand attributes, units, and systems of measurement	recognize and describe spatial relationships	describe and draw geometric shapes from recalled mental images	use geometric models to represent and explain numerical and algebraic relationships	visualize three-dimensional objects and spaces from different perspectives
	use spatial orientation to navigate to the same point from several different starting positions	develop awareness of measurements as approximations, and understand how the tools used to measure affect the level of precision	select appropriate units and scale to estimate and measure angles, perimeter, area, surface area, and volume	select an appropriate unit of measurement or scale, and understand the effects of the choices that are made
STANDARD 4 Measurement 1. Understand attributes, units, and systems of measurement	recognize the attributes of length, capacity, weight, area, and time	understand the need for uniform units of measurement, and develop facility in using the common units of the English and metric systems of measurement	understand both metric and customary systems of measurement, including relationship among units of the same system	analyze how changes in the measurement of one attribute of an object relate to others, such as how the change in the radius or height of a cylinder affects the surface area or volume of the cylinder
	compare and order objects qualitatively by these attributes	identify attributes such as length, area, and volume, and know the type of unit and tool needed to measure each attribute	understand rate of change as a quotient of two different measures	understand rate of change as a quotient of two different measures
make and use measurements in natural situations				

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 4.1 continued	develop referents for estimation	carry out simple unit conversions within a system of measurement (e.g., centimeters to meters, hours to minutes)		use successive approximations to find areas and instantaneous rates of change
	develop a sense of the unit (e.g., length, area) through estimation	explore the effect on size when objects change under simple transformations (e.g., do different shapes made from linking eight cubes have the same surface area? the same volume?)		
	use tools such as rulers to measure	select and apply appropriate standard units (metric and English) and tools (e.g., rulers, grid paper, graduated cylinders) to estimate and measure length, area, volume, weight, time, temperature, and angles;	be proficient in measuring angles in plane figures	apply scaling techniques to view a problem from different perspectives, such as window changes in the graphs of functions
	measure with same size unit (nonstandard and standard)	determine the perimeter, area, and volume of shapes and solids by counting segments, square units, or cubic units	develop and use formulas for the perimeter and area of parallelograms, trapezoids, circles, and simple composite figures	use radian and degree measures
STANDARD 4 Measurement 2. Apply a variety of techniques, tools, and formulas for determining measurements	use repetition of units (iteration) to measure length and area	develop, understand, and use formulas to find the perimeter and area of rectangles	select techniques and tools to measure accurately, with levels of precision appropriate to the situation	understand and apply the concepts of variance and standard deviation as measures of spread in a distribution
		estimate measurements of physical objects by comparing the objects to benchmarks (e.g., estimate the size of an angle by relating it to a right angle)	use ratios and proportions to solve problems involving scale factors	use dimensional analysis for unit conversion and to verify that expressions and equations make sense
		determine the surface area of cubes and other rectangular solids by considering each face	determine an appropriate scale and use scale drawings or models in applications	determine precision, accuracy and measurement errors; identify sources (measurement or round-off errors) and magnitudes of possible errors in a measurement setting; understand how errors propagate within computations; determine how much imprecision is reasonable for various measurements

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 4.2 continued		use map scales to measure distance between locations and make simple scale drawings using grid paper	solve formulas for determining measurements simple problems involving rates and derived measure (e.g., miles per hour)	use successive approximations to illustrate and use the formulas for the volume of a sphere, a general cylinder, and a cone informally apply limit concepts to further develop the concepts of area and instantaneous rate of change
STANDARD 5 Data Analysis, Statistics, and Probability 1. Pose questions and collect, organize, and represent data to answer those questions	gather data about themselves and their surroundings to answer questions that involve multiple responses sort and classify objects and organize data according to attributes of the objects represent data to convey results at a glance, using concrete objects, pictures and numbers	formulate questions they want to investigate design data investigations to address a question collect data using observations, measurement, surveys, or experiments	design experiments and surveys, and consider potential sources of bias in design and data collection recognize types of data (e.g., categorical, count, continuous or measurement, and organize collections of data) choose, create and utilize various graphical representations of data (line plots, bar graphs, stem-and-leaf plots, histograms, scatter plots, circle graphs, and box-and-whisker plots) appropriately and effectively	combine measurements (e.g., length, time, mass, area, volume), using ratios to produce measures such as acceleration, velocity, pressure, and density, as well as dimensionless measures such as trigonometric ratios combine measures (e.g. mass, acceleration, distance) using multiplication to produce measures such as force, work, and person-hours design and carry out appropriate methods for gathering univariate data, both to study the distribution of a variable in one population and to compare the distributions of the same variable in two different populations design appropriate methods for collecting, recording, and organizing data to obtain bivariate data in order to study the association between two variables select appropriate graphical representations and numerical summaries of data

STANDARD 5.1 continued	PRE-K – 8	organize data using tables and graphs (e.g., bar graph, line plot, stem-and-leaf plot, circle graph, and line graph)	6 – 8	9 – 12
		use graphs to analyze data and to present information to an audience		use calculators and computer applications (e.g., spreadsheets, simulation software, and statistical software) appropriately to assist in data collection, organization, and representation
		compare data representations to determine which aspects of the data they highlight or obscure		
STANDARD 5 Data Analysis, Statistics, and Probability		describe parts of the data and the data as a whole		compute, identify, and interpret measures of center and spread (e.g., range, variance, and standard deviation, and interquartile range)
		describe the shape and important features of a set of numerical data, including its range, where the data are concentrated or sparse, and whether there are outliers	find, describe, and interpret mean, median, and mode as measures of the center of a data set; know which measure is best to use in particular situations; and understand how each does and does not represent the data	
		describe the center of sets of numerical data, first informally, then using the median	describe and interpret the spread of a set of data using tools such as range, interquartile range, and box-and-whiskers graphs	describe shapes of one- and two-dimensional data sets
2. Interpret data using methods of exploratory data analysis		classify and describe categorical data (e.g., ways we travel to school) in different ways; analyze and compare the information highlighted by different classifications	interpret graphical representations of data, including description and discussion of the meaning of the shape and features of the graph, such as symmetry, skewness, and outliers	look for symmetry and skewness, clusters and gaps, and possible outliers in data and consider their effects on the interpretation of the data
		compare related data sets, with emphasis on the range, center, and how the data are distributed	analyze associations between variables by comparing the centers, spreads, and graphical representations of related data set	recognize how sample size or transformations of data affect shape, center, and spread

	PRE-K – 8	3-5	6 – 8	9 - 12
STANDARD 5.2 continued		propose and justify conclusions based on data	examine and interpret relationships between two variables, using tools such as scatter plots and approximate lines of best fit	use a variety of representations of data, including scatterplots, frequency distributions, and two-way tables
		formulate questions or hypotheses based on initial data collection, and design further studies to explore them		be able to recognize trends in bivariate data, visually and numerically, and use technology to determine how well different models (e.g., linear, exponential, and quadratic) fit data, while understanding that a perfect fit is unlikely for empirical data
STANDARD 5 Data Analysis, Statistics, and Probability 3. 3. Develop and evaluate inferences, predictions, and arguments that are based on data		describe how data collection methods can impact the nature of the data set	develop conclusions about a characteristic in the population from a well-constructed sample	understand the elements involved in finding good models for phenomena
		discuss the concept of representativeness of a sample within the context of a particular example (e.g., is our class representative of other fifth-grade classes in our town? in the U.S.? in Canada? Why or why not?)	through simulations, develop an understanding about when differences in data may indicate an actual difference in the populations from which the data were collected and when the difference may result from natural variation in samples	apply well-fitting models to predict unobserved outcomes
		compare the data from one sample to other samples and consider why there is variability	use data to answer the questions that were posed, understand the limitations of those answers, and pose new questions that arise from the data	evaluate conclusions based on data
		in simple experiments, infer the structure of the population through drawing repeated samples (with replacements)		use data from samples to estimate population statistics
				use and interpret the normal and binomial distributions appropriately

STANDARD 5 Data Analysis, Statistics, and Probability 4. Understand and apply basic notions of chance and probability	PRE-K – 8 understand notions such as certain, impossible, more likely, less likely	3-5 discuss events as likely or unlikely, and give descriptions of the degree of likelihood in informal terms (e.g., unlikely, very unlikely, certain, impossible) estimate, describe, and test probabilities of outcomes by associating the degree of certainty with a value ranging from 0 to 1 (e.g., in simple experiments involving spinners with different fractions shaded)	6 – 8 make judgments about the likelihood of uncertain events, and be able to connect those judgments to percents or proportions understand what it means for events to be equally likely and for a game or process to be fair compute simple probabilities using appropriate methods, such as lists, tree diagrams, or area models identify complementary, mutually exclusive, independent, and dependent events, and understand how these relationships affect the determination of probabilities	9 - 12 understand and compute probabilities of independent, disjoint, and conditional events understand that some phenomena are random, and apply the law of large numbers to predict long-term behavior use probability distributions to compute probabilities of events

THE PROCESS STANDARDS

STANDARD 6 Problem Solving	STANDARD 7 Reasoning & Proof	STANDARD 8 Communication	STANDARD 9 Connections	STANDARD 10 Representation
<ol style="list-style-type: none"> Build new mathematical knowledge through their work with problems Develop a disposition to formulate, represent, abstract and generalize in situations within and outside mathematics Apply a wide variety of strategies to solve problems and adapt the strategies to new situations Monitor and reflect on their mathematical thinking and solving problems 	<ol style="list-style-type: none"> Recognize reasoning and proof as essential and powerful parts of mathematics Make and investigate mathematical conjectures Develop and evaluate mathematical arguments and proofs Select and use various types of reasoning and methods of proof as appropriate 	<ol style="list-style-type: none"> Organize and consolidate their mathematical thinking to communicate with others Express mathematical ideas coherently and clearly to peers, teachers, and others Extend their mathematical knowledge by considering the thinking and strategies of others Use the language of mathematics as a precise means of mathematical expression 	<ol style="list-style-type: none"> Recognize and use connections among different mathematics ideas Understand how mathematical ideas build on one another to produce a coherent whole Recognize, use, and learn about mathematics in contexts outside of mathematics 	<ol style="list-style-type: none"> Create and use representations to organize, record, and communicate mathematical ideas Develop a repertoire of mathematical representations that can be used purposefully, flexibly, and appropriately Use representations to model and interpret physical, social, and mathematical phenomena

About the Author

Andrea V. Anderson is Research Faculty at the University of Washington, Bothell. Her two primary areas of research are teacher education and informal learning. She received her Ph.D. in Curriculum and Instruction from the University of Washington, Seattle, in 1991 and joined the Association of Science-Technology Centers as the Director of the Teacher Educators Network. Drawing upon her early years as a classroom teacher, followed by curriculum development and teacher education at the Pacific Science Center, she launched the Inquiry Institutes at Science Centers program. She also provided professional development opportunities for science center educators through a Carnegie-funded minigrant program and had oversight for the publication of a major report to the field: *An Invisible Infrastructure, Institutions of Informal Science Education*.

About the Consultant

Virginia Thompson holds a Masters degree in Mathematical Statistics from the University of California, Berkeley. She is a certified California Standard Life teacher in Elementary and Secondary Mathematics, Physics, and Biological Sciences and in Junior College Statistics and Mathematics. Virginia has taught mathematics at the elementary school and university levels. She has been a mathematical editor and coauthored *Building Confidence in Math*. Virginia is retired from the University of California, Lawrence Hall of Science, where she was the Coordinator of Public Programs/Mathematics Specialist and Acting Director of EQUALS. During her tenure at the Lawrence Hall of Science, she developed, implemented, and presented FAMILY MATH inservice programs in English and Spanish and served as the Director of FAMILY MATH.

